M-fivebranes wrapped on supersymmetric cycles

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We construct supergravity solutions dual to the twisted field theories arising when M-theory fivebranes wrap general supersymmetric cycles. The solutions are constructed in maximal $D=7$ gauged supergravity and then uplifted to $D=11$. Our analysis covers Kähler, special Lagrangian and exceptional calibrated cycles. The metrics on the cycles are Einstein, but do not necessarily have constant curvature. We find many new examples of AdS/CFT duality, corresponding to the IR superconformal fixed points of the twisted field theories.

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I. INTRODUCTION

The supergravity duals of the twisted field theories arising on branes wrapping supersymmetric cycles [1] have recently been investigated in [2–5] (for related solutions see [6–10]). The cases that have been considered include fivebranes and D3-branes wrapping holomorphic curves [2,3], and fivebranes [4] and D3-branes [5] wrapping associative three-cycles. Here we will extend these investigations by analyzing M-fivebranes wrapping all other supersymmetric cycles. The new cases covered here are Kähler four-cycles, special Lagrangian three-, four-, and five-cycles, co-associative four-cycles and Cayley four-cycles.

As in previous work it will be convenient to first construct the solutions in $D=7$ gauged supergravity and then uplift to obtain the $D=11$ solutions. Rather than working with different truncated versions of gauged supergravity we will present a unified treatment by working directly with the maximal $SO(5)$ gauged supergravity of [11]. We then employ the results of [12,13] to uplift to $D=11$. This approach has the virtue of highlighting the universal aspects of the various supergravity solutions.

An ingredient in the supergravity solutions will be a metric on the supersymmetric cycle Σ_d. This metric is required to be Einstein, satisfying $R_{ij}=lg_{ij}$, where, factoring out the overall scale of Σ_d, we have $l=0,\pm 1$. The metric on the special Lagrangian cycles will be further restricted to have constant curvature. For the exceptional four-cycles we impose that the metrics are half-conformally flat; i.e., the Weyl tensor is self-dual. For the Kähler cycles it is sufficient that the metric is Kähler-Einstein. Setting $l=-1$, for all cases except for Kähler four-cycles in Calabi-Yau three-folds, we find explicit solutions of the form $\mathrm{AdS}_{7-d}\times\Sigma_d$. These solutions are the gravity duals of the superconformal theories arising on the wrapped brane. For the single case of SLAG five-cycles we also find an exact solution with $l=1$ of the form $\mathrm{AdS}_2\times S^5$.

We begin in Sec. II by analyzing the general aspects of the Bogomol'nyi-Prasad-Sommerfield (BPS) equations arising from $D=7$ gauged supergravity. This is followed in Secs. III–V with a discussion of the BPS equations for the different cases as well as a presentation of the $\mathrm{AdS}_{7-d}\times\Sigma_d$ solutions and the formulas to uplift to $D=11$. Section VI contains some numerical analysis of the BPS equations where we demonstrate the flows when $l=-1$ from an AdS_7 type regions to the $\mathrm{AdS}_{7-d}\times\Sigma_d$ solutions. We also analyze the BPS equations and the singularities of the general flows with $l=\pm 1$. Section VII briefly concludes.

II. MAXIMAL $D=7$ GAUGED SUPERGRAVITY

The Lagrangian for the bosonic fields of maximal gauged supergravity in $D=7$ is given by [11]

$$2\mathcal{L}=e\bigg[R+\frac{1}{2}m^2(T^2-2T_{ij}T^{ij})-P_{\mu ij}P^{\mu ij}-\frac{1}{2}(\Pi_A{}^i\Pi_B{}^jF^{AB}_{\mu\nu})^2-m^2(\Pi^{-1}{}_i{}^AS_{\mu\nu\rho,A})^2\bigg]-6m\,\delta^{AB}S_A\wedge F_B+\sqrt{3}\epsilon_{ABCDE}\delta^{AG}S_G\wedge F^{BC}\wedge F^{DE}+\frac{1}{8m}(2\Omega_5[B]-\Omega_3[B]). \quad (2.1)$$

Here $A,B=1,\ldots,5$ denote indices of the $SO(5)_g$ gauge group, while $i,j=1,\ldots,5$ denote indices of the $SO(5)_c$ local composite gauge group, which are raised and lowered with δ^{ij} and δ_{ij}. The 14 scalar fields $\Pi_A{}^i$ are given by the coset $SL(5,\mathbb{R})/SO(5)_c$ and transform as a **5** under both $SO(5)_g$ (from the left) and $SO(5)_c$ (from the right). The scalar kinetic term, $P_{\mu ij}$, and the $SO(5)_c$ composite gauge field, $Q_{\mu ij}$, are defined as the symmetric and antisymmetric parts of $(\Pi^{-1})_i{}^A(\delta_A{}^B\partial_\mu+gB_{\mu,A}{}^B)\Pi_B{}^k\delta_{kj}$, respectively. Here B^{AB} are the $SO(5)_g$ gauge fields with field strength $F^{AB}=\delta^{AC}F_C{}^B$, and note that the gauge coupling constant is given by $g=2m$. The four-form field strength F_A for the three-form S_A is the covariant derivative $F_A=dS_A+gB_A{}^BS_B$. The potential terms for the scalar fields are expressed in terms of $T_{ij}=\Pi^{-1}{}_i{}^A\Pi^{-1}{}_j{}^B\delta_{AB}$ with $T=\delta^{ij}T_{ij}$. Finally, $\Omega_3[B]$ and $\Omega_5[B]$ are Chern-Simons forms for the gauge fields B that will not play a role in this paper.

The supersymmetry transformations of the fermions are given by

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$$\delta\psi_\mu = \nabla_\mu\epsilon + \frac{1}{20} m T \gamma_\mu \epsilon - \frac{1}{40}(\gamma_\mu{}^{\nu\rho} - 8\delta_\mu{}^\nu\gamma^\rho)\Gamma_{ij}\epsilon\Pi_A{}^i\Pi_B{}^j F^{AB}_{\nu\rho} + \frac{m}{10\sqrt{3}}\left(\gamma_\mu{}^{\nu\rho\sigma} - \frac{9}{2}\delta_\mu{}^\nu\gamma^{\rho\sigma}\right)\Gamma^i\epsilon\Pi^{-1}{}_i{}^A S_{\nu\rho\sigma,A}$$

$$\delta\lambda_i = \frac{1}{2}\gamma^\mu\Gamma^j\epsilon P_{\mu ij} + \frac{1}{2}m\left(T_{ij} - \frac{1}{5}T\delta_{ij}\right)\Gamma^j\epsilon + \frac{1}{16}\gamma^{\mu\nu}\left(\Gamma_{kl}\Gamma_i - \frac{1}{5}\Gamma_i\Gamma_{kl}\right)\epsilon\Pi_A{}^k\Pi_B{}^l F^{AB}_{\mu\nu} + \frac{m}{20\sqrt{3}}\gamma^{\mu\nu\rho}(\Gamma_i{}^j - 4\delta_i^j)\epsilon\Pi^{-1}{}_j{}^A S_{\mu\nu\rho,A}. \tag{2.2}$$

Here γ^μ are the $D=7$ gamma matrices, while Γ^i are those for $SO(5)_c$. Note that $\Gamma^i\lambda_i=0$. Since ϵ is a spinor under $SO(5)_c$, the derivative $\nabla_\mu\epsilon$ has both a spin and an $SO(5)_c$ connection:

$$\nabla_\mu\epsilon = \left(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} + \frac{1}{4}Q_{\mu ij}\Gamma^{ij}\right)\epsilon. \tag{2.3}$$

In order to construct dual supersymmetric solutions corresponding to branes wrapping various supersymmetric cycles, we consider a metric ansatz of the form

$$ds^2 = e^{2f}[d\xi^2 + dr^2] + e^{2g}(d\overline{s_d^2}). \tag{2.4}$$

Here $d\overline{s_d^2}$ is the metric on the supersymmetric d-cycle, Σ_d. We will use a,b to denote tangent space indices on Σ_d. The coordinates ξ^i, $i=0,\ldots,5-d$ span the unwrapped part of the brane with $d\xi^2 \equiv \eta_{ij}d\xi^i d\xi^j = ds^2(\mathbb{R}^{1,5-d})$. The functions f and g depend on the radial coordinate r only.

The solutions we are interested in have an asymptotic region with $e^{2f}\approx e^{2g}\approx 1/r^2$, for small r, corresponding to an AdS_7-type region with the slices of constant r given by $\mathbb{R}^{1,5-d}\times\Sigma_d$, rather than $\mathbb{R}^{1,5}$. This asymptotic region is interpreted as specifying the UV behavior of the field theory corresponding to the wrapped fivebrane. The behavior of the solution in the interior then specifies the IR behavior. In all but one case we find an exact solution of our BPS equations with g constant and $e^{2f}\approx 1/r^2$ corresponding to an $AdS_{(7-d)}\times\Sigma_d$ solution. These solutions are the supergravity duals of the superconformal theories arising on the wrapped fivebrane. We will also numerically exhibit flows from the UV AdS_7 region to the $AdS_{(7-d)}\times\Sigma_d$ IR fixed point.

The $SO(5)$ gauge fields for the supergravity solutions are specified by the spin connection of the metric on Σ_d corresponding to the fact that the theory on the M-fivebrane is twisted. In general, we will decompose the $SO(5)$ symmetry into $SO(p)\times SO(q)$ with $p+q=5$, and excite the gauge fields in the $SO(p)$ subgroup. We will denote these directions by $m,n=1,\ldots,p$. The precise form in each case will be given below. Geometrically, in 11 dimensions, the fivebrane is embedded on a cycle Σ_d within a supersymmetric manifold M. This decomposition corresponds to dividing the directions transverse to the brane into p directions within M and q directions perpendicular to M. In keeping with this decomposition, the solutions that we consider will have a single scalar field excited. More precisely we have

$$\Pi_A{}^i = \mathrm{diag}(e^{q\lambda},\ldots,e^{q\lambda},e^{-p\lambda},\ldots,e^{-p\lambda}) \tag{2.5}$$

where we have p followed by q entries. Note that this implies that the composite gauge field Q is then determined by the gauge fields via $Q^{ij}=2mB^{ij}$.

For the SLAG five-cycle and most of the four-cycle cases the three-form S is non-vanishing. The S equation of motion is

$$m^2\delta_{AC}\Pi^{-1}{}_i{}^C\Pi^{-1}{}_i{}^B S_B = -m{*}F_A + \frac{1}{4\sqrt{3}}\epsilon_{ABCDE}{*}(F^{BC}\wedge F^{DE}) \tag{2.6}$$

and we note that our solutions will have vanishing four-form field strength F_A.

By substituting this kind of ansatz into the supersymmetry variations (2.2) and imposing appropriate projections on the spinor parameters we will then deduce the BPS equations. In the derivation one finds that it is necessary to twist the gauge connection by the spin connection, so that

$$(\overline{\omega}^{bc}\gamma_{bc} + 2mB^{mn}\Gamma_{mn})\epsilon = 0 \tag{2.7}$$

where $\overline{\omega}^{bc}$ is the spin connection one-form of the cycle. Essentially, this is in order to set to zero in Eq. (2.2) the covariant derivative (2.3) in the cycle directions. After imposing the projections on ϵ we are led to identify the appropriate part of the spin connection of the cycle with the appropriate $SO(5)$ gauge fields. In other words, the twisting is dictated by the projections defining the preserved supersymmetry.

In all cases one finds that, in order to satisfy the BPS equations, one has the conditions

$$\gamma^b\Gamma_{mn}F^{mn}_{ab}\epsilon = \frac{\overline{R}}{dm}e^{-2g}\gamma_a\epsilon,$$

$$\gamma^{ab}\Gamma_n F^{mn}_{ab}\epsilon = \frac{\overline{R}}{pm}e^{-2g}\Gamma^m\epsilon, \tag{2.8}$$

where $\overline{R}$ is a constant. Using the relation (2.7) it is easy to show, from the first condition, that the metric on the cycle is necessarily Einstein:

$$\overline{R}_{ab} = l\overline{g}_{ab} \tag{2.9}$$

and so the constant $\overline{R}$ in Eq. (2.8) is precisely the Ricci scalar $\overline{R}=ld$. Given the factor of e^{2g} in Eq. (2.4), we can rescale $\overline{g}_{ab}$ so that $l=0,\pm 1$. Recall that for $d>3$ the Einstein condition implies that the Riemann tensor can be written as

$$\overline{R}_{abcd}=\overline{C}_{abcd}+\frac{2l}{d-1}\overline{g}_{a[c}\overline{g}_{d]b} \tag{2.10}$$

where $\overline{C}$ is the Weyl tensor. For the examples studied previously, the cycles have been two or three dimensional and hence the Einstein condition implies constant curvature, i.e. the Riemann tensor is given by Eq. (2.10) with $\overline{C}=0$. For the four- and five-cycles it is only necessary that the part of the spin connection involved in the gauging have constant curvature. We will return to this point and it will be useful to refer the Einstein equations which we record here:

$$\begin{aligned} R_{\mu\nu}=&P_\mu P_\nu+(\Pi\Pi F)_{\mu\rho}(\Pi\Pi F)_\nu{}^\rho\\ &+3m^2(\Pi^{-1}S)_{\mu\rho\sigma}(\Pi^{-1}S)_\nu{}^{\rho\sigma}-\frac{1}{10}g_{\mu\nu}\\ &\times[m^2(T^2-2T_{ij}T^{ij})+(\Pi\Pi F)^2+4m^2(\Pi^{-1}S)^2] \end{aligned} \tag{2.11}$$

where contractions over $SO(5)_c$, $SO(5)_g$ and spacetime indices are implicit.

III. SPECIAL LAGRANGIAN CYCLES

Let us first consider fivebranes wrapping special Lagrangian (SLAG) 3-, 4- and 5-cycles in Calabi-Yau 3-, 4- and 5-folds, respectively. The dimension p of the transverse space to the fivebrane within the Calabi-Yau manifold is the same as the dimension of the cycle d. Thus both the holonomy group and the structure group of the normal bundle of SLAG d-cycles are $SO(d)$. The appropriate twisting for such wrappings is obtained by simply identifying the whole of the $SO(d)$ spin connection with an $SO(d)$ part of the R symmetry via the splitting $SO(5)\rightarrow SO(d)\times SO(5-d)$.

This twisting can be seen explicitly by considering the supersymmetry preserved by fivebranes wrapping the d-cycles. The relevant projections in $D=11$ were written down, for example, in Sec. 4.2 of [14]. In the language of gauged supergravity we thus impose (in tangent frame)

$$\begin{aligned} \gamma^r\epsilon&=\epsilon\\ \gamma^{ab}\epsilon&=-\Gamma^{ab}\epsilon \end{aligned} \tag{3.1}$$

where $a,b=1,\ldots,d$ label the directions on the cycle. The first condition, which is present in all cases, projects the supersymmetry onto a definite helicity on the fivebrane. The second conditions describe the twisting, implying that, to satisfy the general condition (2.7) that arises in deriving the BPS equations, one simply sets

$$\overline{\omega}_{ab}=2mB_{ab} \tag{3.2}$$

where B_{ab} generate $SO(p)\subset SO(5)$ and we set all other gauge fields to zero. Similarly using the projections in the condition (2.8) one can see explicitly that the metric on the cycle is indeed Einstein type Eq. (2.9).

Let us now discuss each case further in turn.

A. SLAG three-cycles

The supersymmetry preserved by a fivebrane wrapping a SLAG three-cycle corresponds to $N=2$ supersymmetry in $D=3$. Indeed after decoupling gravity and considering scales much smaller than the inverse size of the cycle we obtain an $N=2$ supersymmetric field theory in $D=3$.

The ansatz for the supergravity BPS solutions is given as follows. The metric is given by (2.4) with $d=3$ where the metric on the three-cycle is Einstein type. In three dimensions this implies that it has constant curvature. The scalars are given by Eq. (2.5) with $p=3$, $q=2$:

$$\Pi_A{}^i=(e^{2\lambda},e^{2\lambda},e^{2\lambda},e^{-3\lambda},e^{-3\lambda}). \tag{3.3}$$

The only non-vanishing gauge fields are taken to be B^{ab}, for $a,b=1,2,3$, and these generate $SO(3)\subset SO(5)$. The projections then imply the twisting (3.2). The three-form equation of motion (2.6) is solved by setting $S_A=0$.

The resulting BPS equations are given by

$$\begin{aligned} e^{-f}f'&=-\frac{m}{10}[3e^{-4\lambda}+2e^{6\lambda}]+\frac{3l}{20m}e^{4\lambda-2g}\\ e^{-f}g'&=-\frac{m}{10}[3e^{-4\lambda}+2e^{6\lambda}]-\frac{7l}{20m}e^{4\lambda-2g}\\ e^{-f}\lambda'&=\frac{m}{5}[e^{6\lambda}-e^{-4\lambda}]+\frac{l}{10m}e^{4\lambda-2g}. \end{aligned} \tag{3.4}$$

It should be noted that in this example and for all the cases to be considered in this paper, the preserved supersymmetry parameters are independent of all coordinates except for their radial dependence which is simply determined by $\delta\psi_r$. In all cases, one finds the simple dependence $\epsilon=e^{f/2}\epsilon_0$ where ϵ_0 is constant. Since the Killing spinors are independent of the coordinates on the cycle we can take arbitrary quotients of the cycle, while preserving supersymmetry.

When the curvature of the three-cycle is negative, $l=-1$, corresponding to a possible quotient of hyperbolic three-space, these equations admit a solution of the form $AdS_4\times\mathbb{H}_3$. Specifically we have

$$\begin{aligned} e^{10\lambda}&=2\\ e^{2g}&=\frac{e^{8\lambda}}{2m^2}\\ e^f&=\frac{e^{4\lambda}}{m}\frac{1}{r}. \end{aligned} \tag{3.5}$$

In fact this solution was first constructed in [15]. Here we can interpret it as the dual supergravity solution corresponding to the superconformal field theory arising when an M-fivebrane wraps a SLAG three-cycle $\mathbb{H}_3$, or a quotient thereof. We will analyze the BPS equations numerically in Sec. IV. We will see there that there are solutions with an AdS_7 region for small r describing the UV physics of the wrapped brane, which flow to large r corresponding to the IR physics. We will exhibit a specific flow to the superconformal fixed point (3.5).

Using the results of [11,12] we can uplift solutions to the BPS equations to give supersymmetric solutions in $D=11$. The metric is given by

$$ds_{11}^2=\Delta^{-2/5}ds_7^2+\frac{1}{m^2}\Delta^{4/5}[e^{4\lambda}DY^aDY^a+e^{-6\lambda}dY^idY^i] \tag{3.6}$$

where

$$DY^a=dY^a+2mB^{ab}Y^b$$
$$\Delta^{-6/5}=e^{-4\lambda}Y^aY^a+e^{6\lambda}Y^iY^i \tag{3.7}$$

where $a=1,2,3$, $i=4,5$ and (Y^a,Y^i) are constrained coordinates on S^4 satisfying $Y^aY^a+Y^iY^i=1$. The expression for the four-form can be found in [11,12].

B. SLAG four-cycles

A fivebrane wrapping a SLAG four-cycle gives rise to (1,1) supersymmetry in $D=2$. The metric is given by Eq. (2.4) with $p=4$, $q=1$ and an Einstein metric on the cycle. From Eq. (2.5) the scalars are now given by

$$\Pi_A{}^i=(e^{\lambda},e^{\lambda},e^{\lambda},e^{\lambda},e^{-4\lambda}). \tag{3.8}$$

The only non-vanishing gauge fields are taken to be B^{ab}, for $a,b=1,\ldots,4$, and these generate $SO(4)\subset SO(5)$. The projections then imply the twisting (3.2).

A new feature for this case is that it is now necessary to switch on the three-form S. We let

$$S_5=-\frac{ce^{-8\lambda-4g}}{64\sqrt{3}m^4}e^0\wedge e^1\wedge e^r \tag{3.9}$$

where

$$c=4m^2e^{4g}\epsilon_{a_1a_2a_3a_4}\epsilon^{b_1b_2b_3b_4}F^{a_1a_2}_{b_1b_2}F^{a_3a_4}_{b_3b_4}$$
$$=\epsilon^{a_1a_2a_3a_4}\epsilon^{b_1b_2b_3b_4}\bar{R}_{a_1a_2b_1b_2}\bar{R}_{a_3a_4b_3b_4} \tag{3.10}$$

where in the second line we have used the relation (3.2) between the gauge field B_{ab} and the spin connection $\bar{\omega}_{ab}$. If c is constant then the four-form F_5 vanishes and the S equation of motion (2.6) is satisfied.

In addition one must also satisfy the Einstein and scalar equations of motion. Our assumption that the metric on Σ_d is Einstein type implies that $\bar{R}_{ab}$ is proportional to $\bar{g}_{ab}$ in the Einstein equations (2.11). The ansatz for the scalars and the three-forms imply that all terms in the right-hand side of Eq. (2.11) are proportional to g_{ab} with the possible exception of the terms quadratic in the field strength of the gauge-fields. Since, by Eq. (3.2), $F_{ab}{}^{cd}$ is proportional to $\bar{R}_{ab}{}^{cd}$, to ensure that Einstein's equations are satisfied we must constrain the Riemann tensor on Σ_d. (An equivalent constraint, requiring that no off-diagonal scalar fields in $\Pi_A{}^i$ are excited, arises from the scalar equation of motion.) To get a consistent solution, we will require that the conformal tensor $\bar{C}_{abcd}$ in the decomposition (2.10) vanish, in which case no problematic terms appear. Given the Einstein condition, this is equivalent to assuming constant curvature, so c now depends only on the curvature l of the cycle, and is given by $c=32l^2/3$.

The resulting BPS equations then have the form

$$e^{-f}f'=-\frac{m}{10}[4e^{-2\lambda}+e^{8\lambda}]+\frac{l}{5m}e^{2\lambda-2g}-\frac{l^2}{10m^3}e^{-4\lambda-4g}$$
$$e^{-f}g'=-\frac{m}{10}[4e^{-2\lambda}+e^{8\lambda}]-\frac{3l}{10m}e^{2\lambda-2g}+\frac{l^2}{15m^3}e^{-4\lambda-4g}$$
$$e^{-f}\lambda'=\frac{m}{5}[e^{8\lambda}-e^{-2\lambda}]+\frac{l}{10m}e^{2\lambda-2g}+\frac{l^2}{30m^3}e^{-4\lambda-4g}. \tag{3.11}$$

If we take the cycle to have constant negative curvature, $l=-1$, we find that the BPS equations admit the $AdS_3\times\mathbb{H}^4$ solution

$$e^{10\lambda}=\frac{3}{2}$$
$$e^{2g}=\frac{e^{-6\lambda}}{m^2}$$
$$e^f=\frac{e^{2\lambda}}{m}\frac{1}{r}. \tag{3.12}$$

The uplifted metric in $D=11$ is now given by

$$ds_{11}^2=\Delta^{-2/5}ds_7^2+\frac{1}{m^2}\Delta^{4/5}[e^{2\lambda}DY^aDY^a+e^{-8\lambda}dY^5dY^5] \tag{3.13}$$

where

$$DY^a=dY^a+2mB^{ab}Y^b$$
$$\Delta^{-6/5}=e^{-2\lambda}Y^aY^a+e^{8\lambda}Y^5Y^5 \tag{3.14}$$

where $a=1,2,3,4$ with $Y^aY^a+Y^5Y^5=5$. The expression for the four-form can be found in [11,12].

C. SLAG five-cycles

A fivebrane wrapping a SLAG five-cycle preserves just one supersymmetry. After decoupling gravity, at low energies we get a quantum mechanical model in $D=1$. For this case $d=p=5$ and all of the $SO(5)$ gauge fields are active, but our ansatz (2.5) implies that all of the scalars to zero:

$$\Pi_A{}^i=\delta_A{}^i. \tag{3.15}$$

All five three-forms are now active and we have

$$S_a = -\frac{ce^{-4g}}{64\sqrt{3}m^4} e^0 \wedge e^r \wedge e^a \tag{3.16}$$

where, given the identification (3.2) of gauge and spin connections,

$$c = \frac{96}{5} m^2 e^{4g} F_{a_1 a_2}{}^{[a_1 a_2} F_{a_3 a_4}{}^{a_3 a_4]} \tag{3.17}$$

$$= \frac{24}{5} \bar{R}_{a_1 a_2}{}^{[a_1 a_2} \bar{R}_{a_3 a_4}{}^{a_3 a_4]}. \tag{3.18}$$

To satisfy the S_A equation of motion, we require c to be constant. As for the four-cycle, this condition and the Einstein's equations (2.11) are satisfied if we set $\bar{C} = 0$ and take the five-cycle to have constant curvature, in which case we have $c = 6l^2$.

The BPS equations are given by

$$e^{-f} f' = -\frac{m}{2} + \frac{l}{4m} e^{-2g} - \frac{9l^2}{32m^3} e^{-4g}$$

$$e^{-f} g' = -\frac{m}{2} - \frac{l}{4m} e^{-2g} + \frac{3l^2}{32m^3} e^{-4g}. \tag{3.19}$$

If we set $l = -1$, we find the $AdS_2 \times \mathbb{H}_5$ solution

$$e^{2g} = \frac{3}{4m^2}$$

$$e^f = \frac{3}{4m} \frac{1}{r}. \tag{3.20}$$

On the other hand if we set $l = 1$ we find the $AdS_2 \times S^5$ solution

$$e^{2g} = \frac{1}{4m^2}$$

$$e^f = \frac{1}{4m} \frac{1}{r}. \tag{3.21}$$

The general solution fo the BPS equations is presented in Sec. VI D. Since the scalars are set to zero, the uplifted $D = 11$ metric takes the simple form

$$ds_{11}^2 = ds_7^2 + \frac{1}{m^2} DY^a DY^a \tag{3.22}$$

where

$$DY^a = dY^a + 2mB^{ab} Y^b \tag{3.23}$$

with $Y^a Y^a = 1$. The expression for the four-form can be found in [11,12].

IV. KÄHLER FOUR-CYCLES

The spin connection of a Kähler-cycle is a $U(2) \approx U(1) \times SU(2)$ connection. The appropriate twisting for a five-brane wrapping a Kähler cycle is to identify the $U(1)$ subgroup of this spin connection with a $U(1)$ subgroup of the $SO(5)$ R symmetry. Which subgroup depends on whether the four-cycle is inside a Calabi-Yau three-fold or a Calabi-Yau four-fold. We now consider each case in turn.

A. Kähler four-cycles in Calabi-Yau three-folds

In the case that the four-cycle is in a Calabi-Yau three-fold, corresponding to (4,0) supersymmetry in $D = 2$, there are two transverse directions to the five-brane within the three-fold, so $p = 2$. Equivalently the normal bundle has $SO(2) = U(1)$ structure group and hence the appropriate identification is such that we split $SO(5) \to SO(2) \times SO(3)$ and identify the $U(1)$ part of the spin connection with $SO(2)$.

We let B^{12} generate this $SO(2)$ and set all other gauge fields to zero. The relevant projections on the supersymmetry parameters can be written

$$\gamma^r \epsilon = \epsilon$$

$$\gamma^{12} \epsilon = \gamma^{34} \epsilon = \Gamma^{12} \epsilon \tag{4.1}$$

in a basis where the non-vanishing components of the Kähler-form on the four-cycle are $J_{12} = J_{34} = 1$. We then find that Eq. (2.7) implies that

$$B^{12} = -\frac{1}{4m} \bar{\omega}_{ab} J^{ab} \tag{4.2}$$

where $a,b = 1, \ldots, 4$ and hence the field strength is given by the projection of the Riemann tensor onto the Ricci-form $\bar{\mathcal{R}}_{ab} \equiv \frac{1}{2} \bar{R}_{abcd} J^{cd}$:

$$F^{12} = -\frac{1}{2m} \bar{\mathcal{R}}. \tag{4.3}$$

Since we have $p = 2, q = 3$, given Eq. (2.5), the scalar fields are taken to be

$$\Pi_A{}^i = (e^{3\lambda}, e^{3\lambda}, e^{-2\lambda}, e^{-2\lambda}, e^{-2\lambda}) \tag{4.4}$$

and we can set the 3-form S to zero.

The derivation of the BPS equations again implies that the metric on the Kähler cycle is Einstein. Note that we then have $\bar{\mathcal{R}}_{ab} = lJ_{ab}$. In this case, no other constraint is placed on the cycle. One might expect that, as in the SLAG case, there is a condition coming from the Einstein equations. For SLAG cycles, the conformal part of the curvature (2.10) was required to vanish. However, since here the gauge fields depend only on the $U(1)$ part of the curvature on the cycle, and this has a vanishing conformal tensor, the stress-energy tensor is necessarily proportional to g_{ab} and no such condition arises.

We obtain the BPS equations

$$e^{-f}f' = -\frac{m}{10}[2e^{-6\lambda}+3e^{4\lambda}] + \frac{l}{5m}e^{6\lambda-2g}$$

$$e^{-f}g' = -\frac{m}{10}[2e^{-6\lambda}+3e^{4\lambda}] - \frac{3l}{10m}e^{6\lambda-2g}$$

$$e^{-f}\lambda' = \frac{m}{5}[e^{4\lambda}-e^{-6\lambda}] + \frac{l}{5m}e^{6\lambda-2g}. \tag{4.5}$$

To look for an $AdS_3\times\Sigma_4$ fixed point we set $g'=\lambda'=0$, but find that we are driven to $\lambda\to\infty$. As for all cases we will numerically investigate these equations in Sec. VI.

The uplifted metric in $D=11$ is now given by

$$ds_{11}^2 = \Delta^{-2/5}ds_7^2 + \frac{1}{m^2}\Delta^{4/5}[e^{6\lambda}DY^aDY^a + e^{-4\lambda}dY^idY^i] \tag{4.6}$$

where

$$DY^a = dY^a + 2mB^{ab}Y^b$$

$$\Delta^{-6/5} = e^{-6\lambda}Y^aY^a + e^{4\lambda}Y^iY^i \tag{4.7}$$

where $a=1,2$, $i=3,4,5$ with $Y^aY^a+Y^iY^i=5$. The expression for the four-form can be found in [11,12].

B. Kähler four-cycles in Calabi-Yau four-folds

When the Kähler four-cycle is in a Calabi-Yau four-fold, corresponding to (2,0) supersymmetry in $D=2$, there are now four directions transverse to the fivebrane within the four-fold, so $p=4$. Equivalently, the normal bundle has $U(2)\subset SO(4)$ structure group. In this case the appropriate identification of the $U(1)$ part of the $U(2)$ spin connection is to break $SO(5)\to SO(4)\to U(2)$ and then identify the $U(1)$ part of the spin connection with the $U(1)$ in $U(2)\approx U(1)\times SU(2)$.

Consequently we take only the $U(1)\subset U(2)$ gauge fields to be non-vanishing: equivalently we take $B^{12}=B^{34}$ with all other components vanishing. We have the projections

$$\gamma^r\epsilon = \epsilon,$$

$$\gamma^{12}\epsilon = \gamma^{34}\epsilon = \Gamma^{12}\epsilon = \Gamma^{34}\epsilon \tag{4.8}$$

corresponding to the obvious non-vanishing components of the Kähler form.

We then find

$$B^{12}+B^{34} = -\frac{1}{4m}\bar{\omega}_{ab}J^{ab} \tag{4.9}$$

where $a,b=1,\ldots,4$, and hence

$$F^{12}+F^{34} = -\frac{1}{2m}\bar{\mathcal{R}}. \tag{4.10}$$

In this case, since $p=4$, $q=1$, the ansatz for the scalars is as in the SLAG four-cycle case (3.8). The ansatz for the 3-form is again as for the SLAG four-cycle case (3.9) but now with

$$c = 4m^2e^{4g}\epsilon_{a_1a_2a_3a_4}\epsilon^{b_1b_2b_3b_4}F^{a_1a_2}_{b_1b_2}F^{a_3a_4}_{b_3b_4}$$
$$= 16l^2 \tag{4.11}$$

where in the second line we have substituted for F^{cd}_{ab} in terms of $\mathcal{R}_{ab}$. As in the previous Kähler case, we do not need to impose any additional constraints on the Kähler-Einstein metric on the four-cycle.

The resulting BPS equations then have the form

$$e^{-f}f' = -\frac{m}{10}[4e^{-2\lambda}+e^{8\lambda}] + \frac{l}{5m}e^{2\lambda-2g} - \frac{3l^2}{20m^3}e^{-4\lambda-4g}$$

$$e^{-f}g' = -\frac{m}{10}[4e^{-2\lambda}+e^{8\lambda}] - \frac{3l}{10m}e^{2\lambda-2g} + \frac{l^2}{10m^3}e^{-4\lambda-4g}$$

$$e^{-f}\lambda' = \frac{m}{5}[e^{8\lambda}-e^{-2\lambda}] + \frac{l}{10m}e^{2\lambda-2g} + \frac{l^2}{20m^3}e^{-4\lambda-4g}. \tag{4.12}$$

If we take the cycle to have constant negative curvature, $l=-1$, we find the $AdS_3\times\Sigma_4$ solution

$$e^{10\lambda} = \frac{4}{3}$$

$$e^{2g} = \frac{e^{-6\lambda}}{m^2}$$

$$e^f = \frac{e^{2\lambda}}{m}\frac{1}{r}. \tag{4.13}$$

Note that the form of the uplifted metric in $D=11$ is the same as for the SLAG four-cycles (3.13),(3.14).

V. EXCEPTIONAL CYCLES

There are three exceptional calibrations: the associative three-cycles and the co-associative four-cycles in manifolds of G_2-holonomy and the Cayley four-cycles in manifolds of $Spin(7)$ holonomy. The supergravity duals of fivebranes wrapping associative three-cycles was considered in [4] and here we will analyze the remaining two cases.

A. Co-associative four-cycles

In this case the four-cycle has an $SO(4)\approx SU(2)_L\times SU(2)_R$ spin connection. Since $p=3$, we split the R symmetry $SO(5)\to SO(3)\times SO(2)$ and the appropriate twisting is obtained by identifying $SU(2)_L$ with $SO(3)$. This twist leads to (2,0) supersymmetry in $D=2$.

A discussion of the appropriate projections can be found in Sec. 4.3 of [14]. Here we write these as

$$\gamma^r \epsilon = \epsilon$$

$$\gamma^+_{ab}\epsilon = 0$$

$$\Gamma^{23}\epsilon = \gamma^-_{12}\epsilon, \quad \Gamma^{31}\epsilon = \gamma^-_{13}\epsilon, \quad \Gamma^{12}\epsilon = \gamma^-_{14}\epsilon \tag{5.1}$$

where the pluses and minuses refer to self-dual and anti-self-dual parts, respectively, and $a,b = 1,\ldots,4$. The $SO(3)$ gauge fields are generated by B^{mn}, $m,n = 1,2,3$, and we set all other gauge fields to zero. From Eq. (2.7) we deduce

$$\bar{\omega}^{-12} = -mB^{23}$$

$$\bar{\omega}^{-13} = -mB^{31}$$

$$\bar{\omega}^{-14} = -mB^{12}. \tag{5.2}$$

Given $p=3$, $q=2$, the scalar ansatz is the same as for the SLAG three-cycles (3.3) and the three-form S can be set to zero. The condition (2.8) again implies that the metric on the cycle is Einstein. In order to ensure that Einstein's equations are solved we note that since, unlike the SLAG case, only the anti-self-dual part of the spin connection on the cycle enters, it is only necessary to set $\bar{C}^- = 0$ in Eq. (2.10). In other words we take the associative four-cycle to have a conformally half-flat Einstein metric. The only compact examples with $l=1$ are S^4 or $\mathbb{C}P^2$ and for $l=0$ we have flat space or $K3$.

The BPS equations are now

$$e^{-f}f' = -\frac{m}{10}[3e^{-4\lambda} + 2e^{6\lambda}] + \frac{l}{5m}e^{4\lambda-2g}$$

$$e^{-f}g' = -\frac{m}{10}[3e^{-4\lambda} + 2e^{6\lambda}] - \frac{3l}{10m}e^{4\lambda-2g}$$

$$e^{-f}\lambda' = \frac{m}{5}[e^{6\lambda} - e^{-4\lambda}] + \frac{2l}{15m}e^{4\lambda-2g}. \tag{5.3}$$

Setting $l=-1$ we find an $\mathrm{AdS}_3 \times \Sigma_4$ solution

$$e^{10\lambda} = 3$$

$$e^{2g} = \frac{e^{8\lambda}}{3m^2}$$

$$e^f = \frac{2e^{4\lambda}}{3m}\frac{1}{r}. \tag{5.4}$$

The uplifted solutions in $D=11$ have the same structure as the SLAG three-cycles (3.6),(3.7).

B. Cayley four-cycles

The four-cycle has an $SO(4) \approx SU(2)_L \times SU(2)_R$ spin connection. Given now $p=4$, we split the R symmetry $SO(5) \to SO(4) \approx SU(2)'_L \times SU(2)'_R$ and the appropriate twisting is obtained by identifying $SU(2)_L$ with $SU(2)'_L$. This twist leads to (1,0) supersymmetry in $D=2$.

Again an explicit discussion of the projections can be found in Sec. 4.3 of [14]. Here we will use

$$\gamma^r \epsilon = \epsilon$$

$$\gamma^+_{ab}\epsilon = \Gamma^+_{ab}\epsilon = 0$$

$$\Gamma^-_{ab}\epsilon = -\gamma^-_{ab}\epsilon. \tag{5.5}$$

The $SU(2)'_L$ gauge fields are generated by B^{-ab}, $a,b = 1,\ldots,4$, and we set all other gauge fields to zero. From Eq. (2.7) we deduce

$$\bar{\omega}^{-ab} = 2mB^{-ab}. \tag{5.6}$$

Since $p=4$, $q=1$, the scalar field ansatz is the same as the SLAG four-cycles and the Kähler four-cycles in Calabi-Yau four-folds (3.8). The three-form S also has the same form (3.9), though, now,

$$c = 4m^2 e^{4g}\epsilon_{a_1a_2a_3a_4}\epsilon^{b_1b_2b_3b_4}F^{a_1a_2}_{b_1b_2}F^{a_3a_4}_{b_3b_4}$$
$$= 4\bar{R}^-_{abcd}\bar{R}^{-abcd}. \tag{5.7}$$

As before, if c is constant, then the S equation of motion is satisfied. As in the co-associative case, this condition is satisfied as are the Einstein equations if we take the cycle to be conformally half-flat by setting $\bar{C}^- = 0$ in Eq. (2.10). We then get $c = 16l^2/3$.

The BPS equations then read

$$e^{-f}f' = -\frac{m}{10}[4e^{-2\lambda} + e^{8\lambda}] + \frac{l}{5m}e^{2\lambda-2g} - \frac{l^2}{20m^3}e^{-4\lambda-4g}$$

$$e^{-f}g' = -\frac{m}{10}[4e^{-2\lambda} + e^{8\lambda}] - \frac{3l}{10m}e^{2\lambda-2g} + \frac{l^2}{30m^3}e^{-4\lambda-4g}$$

$$e^{-f}\lambda' = \frac{m}{5}[e^{8\lambda} - e^{-2\lambda}] + \frac{l}{10m}e^{2\lambda-2g} + \frac{l^2}{60m^3}e^{-4\lambda-4g}. \tag{5.8}$$

If we set $l=-1$, we find the following $\mathrm{AdS}_3 \times \Sigma_4$ solution:

$$e^{10\lambda} = \frac{12}{7}$$

$$e^{2g} = \frac{e^{-6\lambda}}{m^2}$$

$$e^f = \frac{e^{2\lambda}}{m}\frac{1}{r}. \tag{5.9}$$

The structure of the uplifted metric in $D=11$ follows the SLAG four-cycle case and is given by Eqs. (3.13) and (3.14).

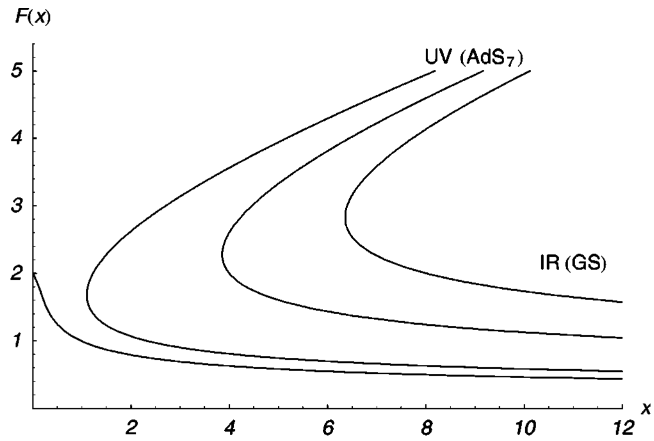

FIG. 1. Behavior of the orbits for co-dimension 2 with $l=-1$. The AdS_7-type UV region is when F and x are both large. The singularity, IR(GS), in the IR region is of the good type.

VI. ANALYZING THE BPS EQUATIONS

To further analyze the BPS equations it is useful to group them via the co-dimension of the cycle.

A. Co-dimension 2

The only co-dimension two-cycle that we have been considering is the Kähler four-cycle in a Calabi-Yau 3-fold. Let us introduce the new variables

$$a^2=e^{2g}e^{-12\lambda}$$

$$e^h=e^{f-6\lambda}. \tag{6.1}$$

The BPS equations are then somewhat simpler:

$$e^{-h}h'=-\frac{m}{2}[3e^{10\lambda}-2]-\frac{l}{ma^2}$$

$$e^{-h}\frac{a'}{a}=-\frac{m}{2}[3e^{10\lambda}-2]-\frac{3l}{2ma^2}$$

$$e^{-h}\lambda'=\frac{m}{5}[e^{10\lambda}-1]+\frac{l}{5ma^2}. \tag{6.2}$$

The analysis is further simplified by introducing $x=a^2$ and $F=x^{2/3}e^{10\lambda}$, giving the ordinary differential equation (ODE)

$$\frac{dF}{dx}=\frac{2m^2F}{[3m^2(3Fx^{1/3}-2x)+9l]}. \tag{6.3}$$

Typical flows in the (F,x) plane for the case of $l=-1$ and $l=1$ are plotted in Figs. 1 and 2, respectively.

When both F and x are large we get $F\approx x^{2/3}(1-2l/m^2x)$. Using a as a radial variable, we find that this gives rise to the asymptotic behavior

$$ds^2=\frac{4}{m^2a^2}da^2+a^2(d\xi^2+\overline{ds_4^2})$$

$$e^{10\lambda}=1-\frac{2l}{m^2a^2}. \tag{6.4}$$

This is precisely what we expect for the wrapped M-fivebrane. The scalars vanish and the metric has the form of AdS_7 except that the slices of constant a have $\mathbb{R}^{1,5}$ replaced with $\mathbb{R}^{1,1}\times\Sigma_4$, where Σ_4 is the four-cycle with a Kähler-Einstein metric. Note that the next to leading order behavior of the scalar field corresponds to the insertion of the boundary operator $\mathcal{O}_4$ of conformal dimension $\Delta=4$, which is dual to an operator constructed from the scalar fields in the M-fivebrane theory.

The IR behavior of the wrapped M-fivebrane is obtained by analyzing the asymptotic behavior of the flows. This case is the exception in that there is not a flow to an IR $AdS_3\times\Sigma_4$ fixed point when $l=-1$. In fact, as one can see from Fig. 1, the flows end up in a region of small F and large x. This limit can be analyzed explicitly. One finds $F\approx 1/x^{1/3}$ with $e^{10\lambda}\approx 1/x$ tending to zero. The asymptotic metric is singular and given by

$$ds^2=\frac{1}{m^2a^{22/5}}da^2+a^{-2/5}(d\xi^2+\overline{ds_d^2}). \tag{6.5}$$

It is straightforward to demonstrate that the (00) component of the uplifted $D=11$ metric (4.6) is bounded as we approach the singularity and hence this is a ''good'' singularity by the criteria of [2].

For $l=1$, one still has the AdS_7-type region at large F and x, but now the flows are different. As can be seen in Fig. 2, there are three possibilities. One can flow to the small F and large x region and one obtains the asymptotic behavior (6.5) with a good singularity. A good singularity is also found for the special orbit with $F=0$ and $x=3l/2m^2$. There are also flows to F constant and $x=0$ which give rise to bad singularities.

Finally, it is probably worth noting that we can in fact integrate Eq. (6.3) to explicitly realize the behavior discussed above. In the original variables one gets the general relation

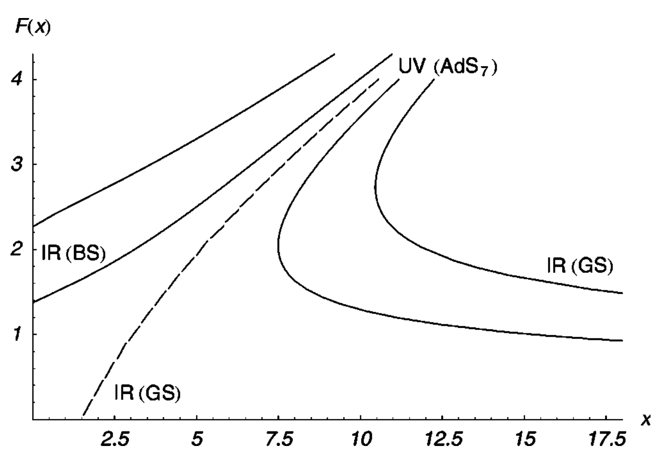

FIG. 2. Behavior of the orbits for co-dimension 2 with $l=1$. IR(GS) and IR(BS) indicate the good and bad singularities in the IR region, respectively.

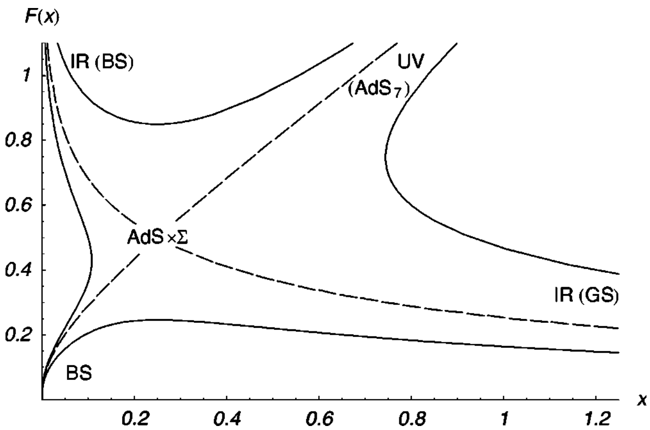

FIG. 3. Behavior of the orbits for co-dimension 3 with $l=-1$. Note the flow from the AdS_7-type region when F,x are large to the IR fixed point and the flows to the good and bad singularities in the IR, IR(GS) and IR(BS), respectively.

$$-\frac{2l^2}{m^4}\ln(m^2e^{2g-2\lambda}+l)+\frac{2l}{m^2}e^{2g-2\lambda}-e^{4g-4\lambda}+e^{4g+6\lambda}=C \tag{6.6}$$

for some constant C.

B. Co-dimension 3

There are two examples with co-dimension 3: the SLAG three-cycles and the co-associative four-cycles. In this case it is useful to introduce the new variables

$$a^2=e^{2g}e^{-8\lambda}$$
$$e^h=e^{f-4\lambda}. \tag{6.7}$$

The BPS equations are then given by

$$e^{-h}h'=-\frac{m}{2}[2e^{10\lambda}-1]-\frac{\gamma}{ma^2}$$

$$e^{-h}\frac{a'}{a}=-\frac{m}{2}[2e^{10\lambda}-1]-\frac{\beta}{ma^2}$$

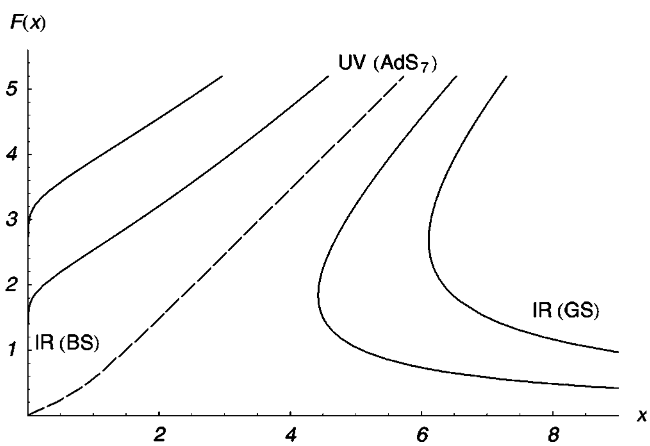

FIG. 4. Behavior of the orbits for co-dimension 3 with $l=1$.

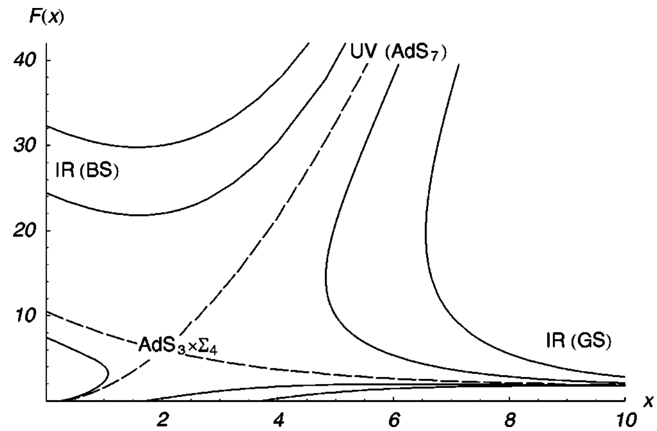

FIG. 5. Behavior of the orbits for co-dimension 4 with $l=-1$. Note the flow from the AdS_7-type region when F,x are large to the IR fixed point and the flows to the good and bad singularities in the IR, IR(GS) and IR(BS), respectively.

$$e^{-h}\lambda'=\frac{m}{5}[e^{10\lambda}-1]+\frac{\alpha}{2ma^2} \tag{6.8}$$

where $(\alpha,\beta,\gamma)=(l/5,3l/4,l/4)$ for the SLAG three-cycles and $(4l/15,5l/6,l/3)$ for the associative four-cycles. We next define $x=a^2$ and $F=xe^{10\lambda}$ and obtain the ODE

$$\frac{dF}{dx}=\frac{F[m^2x-5\alpha+2\beta]}{x[m^2(2F-x)+2\beta]}. \tag{6.9}$$

The typical behavior of $F(x)$ is illustrated in Fig. 3 for $l=-1$ and Fig. 4 for $l=1$. The region where both x and F large corresponds to the AdS_7-type region describing the UV behavior of the wrapped brane. We have $F\approx x-5\alpha/m^2$ and using a as a radial variable we obtain the asymptotic behavior

$$ds^2=\frac{4}{m^2a^2}da^2+a^2(d\xi^2+d\overline{s_d^2})$$

$$e^{10\lambda}=1-\frac{5\alpha}{m^2a^2}. \tag{6.10}$$

Again we see that the operator $\mathcal{O}_4$ is switched on.

For $l=-1$ we can flow from the UV region to the $AdS\times\Sigma_d$ fixed point that was given in Eqs. (3.5) and (5.4) for the SLAG three-cycles and co-associative cycles, respectively. There are also flows exhibited in Fig. 3 which flow to small F for large x. These behave like $F\approx 1/x$ with $e^{10\lambda}\approx 1/x^2$ tending to zero. The asymptotic metric is given by

$$ds^2=\frac{4}{m^2a^{26/5}}da^2+a^{-6/5}(d\xi^2+d\overline{s_d^2}). \tag{6.11}$$

It is straightforward to demonstrate that these are good singularities. There are also flows from the AdS_7 region to large F and small x. They have $F\approx[(2\beta-5\alpha)/2m^2]\ln x$ and give rise to bad singularities. Similarly the flow from the AdS_3 fixed point to small F and x have $F\approx x^{(2\beta-5\alpha)/2\beta}$ and give bad singularities.

When $l=1$ the flows from the UV to the IR are illustrated in Fig. 4. The flows to small F and large x give rise to the asymptotic behavior (6.11) and hence have good singularities. The singularities for the flows to small F and x are the same as for $l=-1$ and hence are bad.

C. Co-dimension 4

There are three examples with co-dimension 4: SLAG four-cycles, Kähler four-cycles in Calabi-Yau four-folds and Cayley four-cycles. It is now convenient to introduce the new variables

$$a^2=e^{2g}e^{-4\lambda}$$

$$e^h=e^{f-2\lambda}. \tag{6.12}$$

The BPS equations are then given by

$$e^{-h}h'=-\frac{m}{2}e^{10\lambda}-\frac{\beta}{2e^{10\lambda}a^4}$$

$$e^{-h}\frac{a'}{a}=-\frac{m}{2}e^{10\lambda}-\frac{\alpha}{2a^2}$$

$$e^{-h}\lambda'=\frac{m}{5}[e^{10\lambda}-1]+\frac{\alpha}{10a^2}+\frac{\beta}{10e^{10\lambda}a^4} \tag{6.13}$$

where $\alpha=l/m$ and $\beta=l^2/3m^3$, $l^2/2m^3$ and $l^2/6m^3$ for the SLAG, Kähler and Cayley four-cycles, respectively.

We next define $x=a^2$ and $F=x^2e^{10\lambda}$ and obtain the ODE

$$\frac{dF}{dx}=\frac{F(\alpha+2mx)-\beta x}{mF+\alpha x}. \tag{6.14}$$

The typical behavior of $F(x)$ is illustrated in Fig. 5 for $l=-1$ and Fig. 6 for $l=1$. The region of x and F large corresponds to the AdS_7-type region describing the UV behavior of the wrapped brane. We have $F\approx x^2-(\alpha/m)x$. Using a as a radial variable we obtain the asymptotic behavior

$$ds^2=\frac{4}{m^2a^2}da^2+a^2(d\xi^2+d\overline{s_d^2})$$

$$e^{10\lambda}=1-\frac{\alpha}{ma^2}. \tag{6.15}$$

The asymptotic behavior of the scalar again indicates that $\mathcal{O}_4$ is switched on.

For $l=-1$ we can flow from the UV region to the AdS $\times\Sigma_4$ fixed points that were given in Eqs. (3.12), (4.13) and (5.9) for the SLAG, Kähler and Cayley four-cycles, respectively. There are also flows exhibited in Fig. 5 which flow to $F=\beta/2m$ for large x. We then have $e^{10\lambda}\approx(\beta/2m)/x^2$ tending to zero. Again it is straightforward to demonstrate that

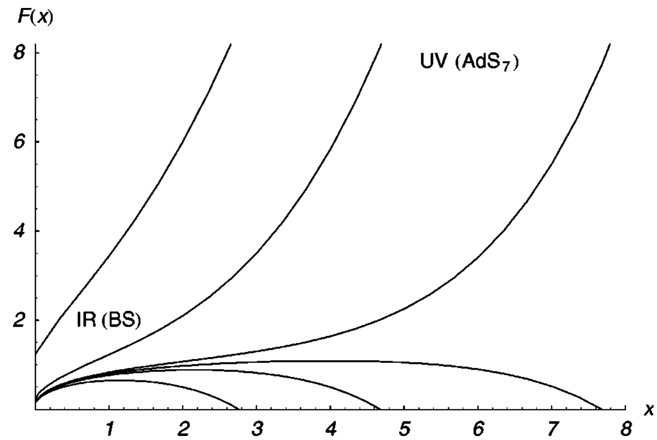

FIG. 6. Behavior of the orbits for co-dimension 4 with $l=1$.

these are good singularities. There are also flows from the AdS_7 region to constant F and small x. The asymptotic metrics have bad singularities.

The behavior of the flows for $l=1$ is illustrated in Fig. 6. The flows from the UV region end up with F constant when $x=0$ and have bad singularities.

We conclude this subsection by determining the central charges of the two dimensional conformal field theories arising at the fixed points of the flows by generalizing the argument of [2]. We use

$$c=\frac{3R_{AdS_3}}{2G_3} \tag{6.16}$$

and relate the three-dimensional Newton's constant G_3 to the eleven-dimensional Newton's constant as in [2]. To do this we work with units where the radius of AdS_7 in the $AdS_7\times S^4$ solution is one by setting $m=2$. We then find

$$c=\frac{8N^3}{\pi^2}\,e^{f_0+4g}\mathrm{Vol}(\overline{\Sigma}) \tag{6.17}$$

where $\mathrm{Vol}(\overline{\Sigma})$ is the volume of the four-cycle and $e^f\equiv e^{f_0}/r$ at the fixed point. From Eqs. (3.12), (4.13), (5.4), and (5.9) we get $e^{f_0+4g}=1/48$, 3/128, 1/48, and 7/384 for the SLAG, Kähler (in four-folds), co-associative, and Cayley four-cycles, respectively.

D. Co-dimension 5

The SLAG five-cycle is the only co-dimension 5 case. It is rather different than the other cases in that the scalars are all set to zero. To solve the BPS equations (3.19) we first introduce a new radial variable ρ via

$$\frac{d\rho}{dr}=e^{2f}. \tag{6.18}$$

We then find the general solution is given by

$$ds^2=-e^{2f}dt^2+e^{-2f}d\rho^2+\rho^2d\overline{s_5^2} \tag{6.19}$$

with

$$e^{2f}=\frac{m^2}{4\rho^6}\left(\rho^2-\frac{l}{4m^2}\right)^2\left(\rho^2+\frac{3l}{4m^2}\right)^2 \tag{6.20}$$

which flows for $l=-1$ or $l=1$ to the conformal fixed points given in Eqs. (3.20) or (3.21), respectively.

VII. DISCUSSION

We have presented a large class of supergravity solutions that are dual to the twisted theories arising on M-fivebranes wrapping general supersymmetric cycles. An Einstein metric on the cycle is an ingredient in the construction: for the SLAG cycles it must have constant curvature, for the Kähler cycles it must be Kähler-Einstein, and for the co-associative and Cayley four-cycles it must be conformally half-flat.

The solutions have an asymptotic AdS_7-type region that describes the UV physics. When the curvature of the Einstein metric on the cycles is negative, $l=-1$, in all but one case—Kähler four-cycles in Calabi-Yau three-folds—there is a flow to an IR fixed point of the form $AdS_{7-d}\times\Sigma_d$. These fixed points are dual to the superconformal field theories arising on the M-fivebrane and thus provide new examples of AdS/CFT duality. For positive curvature, $l=1$, we only found such a fixed point for SLAG five-spheres. We also exhibited flows to other IR limits and determined whether the resulting singularities were of a good or bad type according to the criteria of [2]. It will be interesting to study all of the IR physics in more detail. When the cycle is Ricci-flat, $l=0$, the cycle can either be flat or for Kähler, co-associative, or for Cayley four-cycles it can also be K3 [if we relax compactness it could be any four manifold with $SU(2)$ holonomy]. In this case the gauge fields are zero and there is no twisting and so we simply have a fivebrane wrapping $\mathbb{T}^4$ or K3, whose supergravity solutions are well known.

The solutions that have been constructed here and in [2,4,5] have the minimal gauge fields active consistent with the required twisting. It would be interesting to generalize our solutions to include more general gauge fields which correspond to cycles with the most general normal bundles. Note, for example, that this would distinguish fivebranes wrapping four-cycles in eight-manifolds with $Sp(2)$ holonomy from those with $SU(4)$. It would also be interesting to try and find solutions that relax the Einstein condition. It is possible that such generalizations will involve activating more than a single scalar field. Another direction to pursue is to construct supergravity solutions corresponding to having both fivebranes and membranes involved. For example, it might be possible to construct supergravity solutions analogous to the configurations that were investigated from the M-fivebrane world-volume point of view in [16,17].

ACKNOWLEDGMENTS

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